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A knowledge based freight management decision support system incorporating economies of scale: multimodal minimum cost flow optimization approach

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Abstract This study developed a framework incorporating economies of scale into the multimodal minimum cost flow problem. To properly account for the economies of scale observed in practice, we explicitly modelled economies of scale on quantity, distance and vehicle size in a given multimodal freight network. The proposed multimodal minimum cost flow problem formulation has concave equations due to economies of scale for quantity, nonlinear equations due to economies of scale for both quantity and distance, and non-continuous equations due to the economies of scale for vehicle size. A genetic algorithm was applied to find acceptable route, mode, and vehicle size choices for the multimodal minimum cost flow problem. We demonstrated how the economies of scale influenced system (mode), route choices, and total cost under various demand/service capacity scenarios. Our results will lead into more realistic assessments of intermodal system by explicitly considering the three types of economies of scale.

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1 Introduction

The multimodal freight system has been recognized as an alternative to the truck-only system [2, 7, 14, 26]. Despite its disadvantages—such as high extra costs for relatively short-distance collection/distribution by trucks and their transhipments, and a less-flexible schedule—the multimodal freight system has great potential to significantly reduce total logistics costs, mainly through economies of scale gained in long-haulage with non-road transport modes [1]. However, it is generally understood that specifying economies of scale within a cost function is quite difficult [16]. In fact, when a logistics optimization problem is formulated in Operations Research (OR) problem, the economies of scale, was almost always simplified rather than fully incorporated. In addition, the main stream of such research efforts was to find the location of hubs or multimodal terminals [20, 22] instead of finding optimal combinations of freight modes/routes. The former is usually known as a network design problem (NDP) while the latter is often called as a Minimum Cost Flow Problem (MCFP). While there have been several attempts to embed the concept of economies of scale in NDP, it has rarely applied for MCFP due to mainly the computational complexity. It is a key challenge that this paper addresses later. Given multiple modes are available, another challenge is to represent multimodality in MCFPs. As the number of transport modes increases at a given network, the solution procedure of MCFPs generally becomes more complicated. If more than two modes are combined in MCFPs, it is called as a Multimodal Minimum Cost Flow Problem (MMCFP).

Reflecting these challenges, this study aims at formulating a MMCFP incorporating economies of scale and at solving it by using a proposed genetic algorithm (GA) based heuristic algorithm. The reason to use GA is explained in detail in Appendices 1 and 2. In essence, traditional linear and non-linear programming methods are not suitable for solving the proposed MMCFP since the objective function consists of non-linear and even non-continuous cost functions.

The expected outcome of this study is mode (system[1]) choice, route choice, and batch strategy[2] (among different sized vehicles) for the given ODs either directly or via hubs. These three separate outcomes (i.e. system choice, route choice, and detailed mode size), which has not been shown in previous studies, would show the feasibility of selecting multiple modes with different sized vehicles between origin and destination nodes. The three outcomes should not be obtained in a framework of 'step-by-step' procedure. Instead, they should be selected simultaneously since one outcome obviously influences the others [21, 27]. When 300 TEUs should be sent from A to B, for example, several combinations in terms of freight modes and each mode's size are considered. Based on the selections of sized vehicles/vessels, the transport cost structure of the mode/system is determined. Once the transport cost of a mode/system is decided, the competiveness among the modes considered is fixed. Thus, the batch strategy (i.e. how to group different sized in a mode/system) influences mode/system choice and further route choice. Also, the changes of mode/route choice make shippers modify the batch strategy as well. Such a consideration combining mode/system choice, route choice, and batch strategy would obviously lead to more realistic assessments in multimodal systems with explicit considerations of economies of scale.

The rest of this paper is organized as follows. In Sect. 2, based on the definition of multimodal freight transport system, some overlooked multimodal network issues are discussed. In Sect. 3, we introduced and defined three types of economies of scale in a freight transport market: Economies of Scale for Quantity (ESQ), Economies of Scale for Distance (ESD), and Economies of Scale for Vehicle Size (ESVS).[3] Based on these three types of economies of scales, the MMCFP is formulated in Sect. 4. The formulated problem is solved in Sect. 5. The final outcome for a hypothetical network is presented: route/mode choice and the batch strategy for the chosen route/modes in terms of combination of different sized vehicles. Finally, concluding remarks section summarized the findings from this study and recommendations for future research.

2 Proposed network representation and route/system choice sets

We begin our discussion by defining a multimodal freight system. The European Conference of Ministers of Transport (ECMT) defined the multimodal (combined) transport system as follows:

Combined transport is a transport in which the major part of the European journey is carried out by ***rail***, ***inland waterways or sea*** and in which any initial and/or final legs carried out by ***roads*** are as short as possible [8].

It is explicitly pointed out that trucks take the initial and final legs (i.e., short distances) and non-road modes serve for the main haulage (i.e., longer distance). Thus, when a multimodal network is drawn in this study, we assume all initial and final trips are made by trucks even though there are some exceptions in practice.

Figure 1a, b describe the proposed multimodal network representation with non-road drayage penalty ($\delta_{initial}$) and extra transhipments ($TSc(truck, rail)$). The penalty is only applicable for some complicated freight chains: we call it "2nd level multimodal systems. This concept is introduced since a truck's short trip should be described when the initial mode is either rail or vessel. More specifically, the penalty concept shown in Fig. 1b leads to more realistic and flexible options such as *truck* → *rail* → *vessel* → *rail* → *truck* rather than a conventional multimodal option such as *truck* → *rail* → *truck*. Based on the proposed hypothetical network representation, Fig. 2 shows an example of arc (1, 3) with nine feasible combinations of transport modes. Obviously, these combinations are applicable to any other OD pairs.

Figure 1 is the visualization of the case in which drayage is either road or rail while Fig. 2 is the enumeration of the proposed freight option considered in this study. It is notable that Figs. 1 and 2 are drawn with same symbols. Thus, the two figures would be read together for better understanding. The freight options indicated as r in Fig. 2 show the feasible combinations; $r = 1$(the truck-only system); $r = 2$ and 6 (conventional rail based- and vessel based multimodal systems, respectively). The others ($r = 3, 4, 5, 7, 8,$ and 9), so-called "2nd level multimodal systems", have at least one rail-drayage at either initial or

[1] The sequence of freight transport *modes* (i.e., multimodal *system*).

[2] The batch strategy is, when several sized vehicles of a transport mode are available, a decision on the sequence of them. (e.g. For shipping 200 TEU, if there are 60, 75, 140 TEU trains are available, a feasible batch strategy is that 140 TEU train is used first and then 60 TEU train is used).

[3] It is obvious ESVS influences the batch strategy, which is a main issue of this paper.

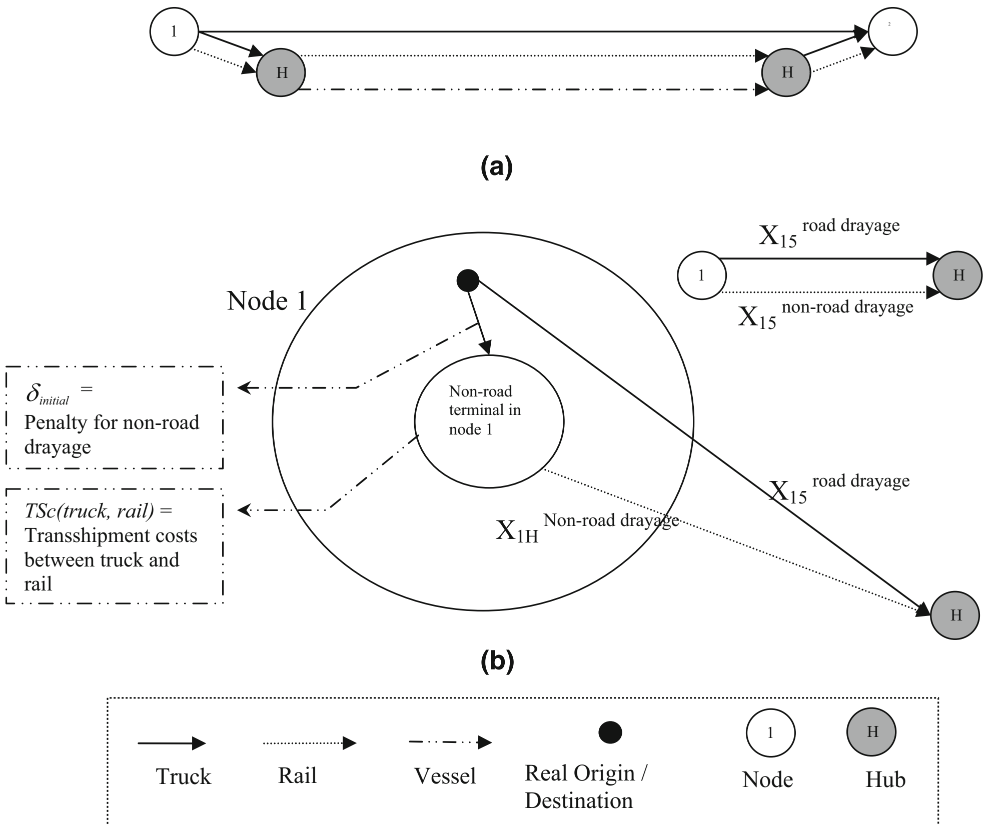


Fig. 1 Multimodal network representation

final leg. The 2nd level multimodal systems (r = 3, 4, 5, 7, 8, and 9) pass the same consecutive nodes (1–5–6–3) to the conventional multimodal systems (r = 2 and 6). It is noted that a penalty is applied for all the 2nd level multimodal systems due to the extra truck drayage described in Fig. 1b. This is because any initial and/or final legs are carried out by roads according to the definition of multimodal transport. To sum up, the proposed multimodal network representation enable to examine more complicated freight modal combination.

3 Previous and proposed unit cost functions incorporating economies of scale

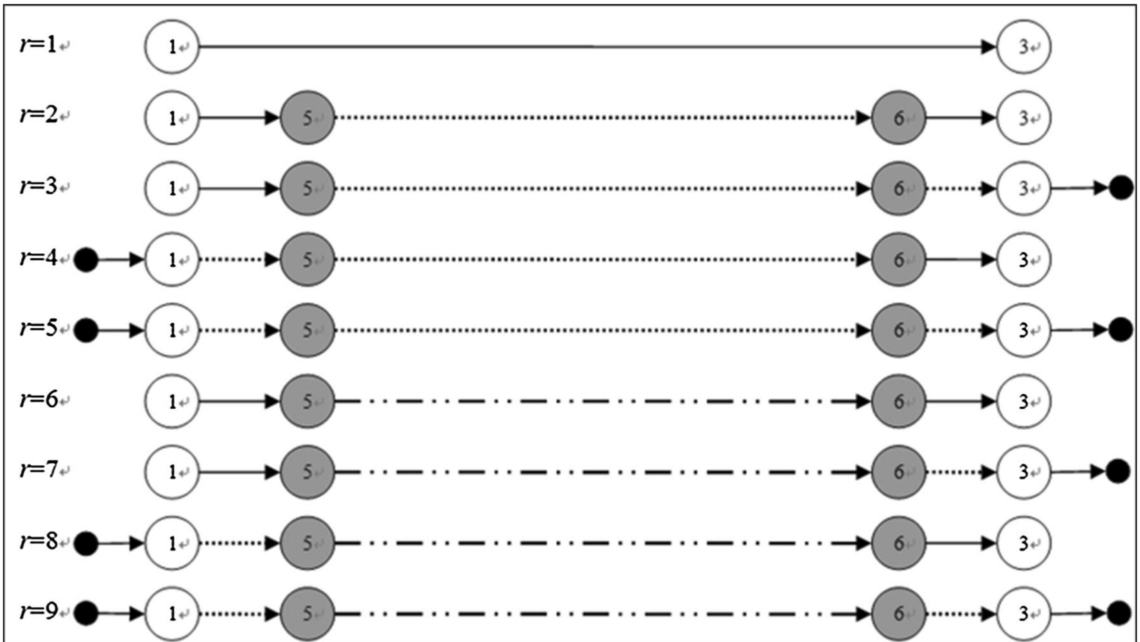
In many MMCFPs, *the unit cost function* is assumed as a linear function that is proportional to quantity (e.g. TEU), distance (e.g. km), or the composite form (e.g. TEU km) for analytical simplicity. A realistic way to specify these three types of economies of scales in a MMCFP would be to develop an enhanced unit cost function that is likely to be a non-linear (even non-continuous) function of quantity, distance, and vehicle size, and to incorporate it into the objective function. Since the total cost in the objective function is determined by multiplying the unit cost function by the assigned quantities (a decision variable in many cases), the unit cost function plays a vital role in MMCFPs.

3.1 Previous unit cost function incorporating economies of scale in OR problem

Since there is very limited number of MCFP incorporating economies of scale, we expanded our scope to Network Design Problem (NDP) and other passenger related operations research (OR) problems in reviewing cost functions in previous studies. When considering ESQ (Economies of Scale in terms of Quantity) in previous studies, the unit cost in the objective function has been simplified to be as

- constant unit costs that are homogeneously applied to every link (arc) and are consequently linearly associated with the quantity assigned to it [24]
- piecewise linear cost functions in which the unit cost is stepped down when the quantity shipped is over a certain assumed quantity criteria (Chang [3], or
- non-linear discount functions that are dependent only on quantity [10, 20, 22].

A typical objective function for a MMCFP is to minimize the total cost, TC_{ij}, as follows:



Options (r)	Mode(s): $k_n \in K$					Description
	Penalty ($\delta_{initial}$)	Pre-haulage (k_1)	Main haulage (k_2)	Post-haulage (k_3)	Penalty (δ_{final})	
1	*0*	*0*	*1*	*0*	*0*	Truck-only system
2	*0*	*1*	*2*	*1*	*0*	Conventional Rail-based intermodal system
3	*0*	*1*	*2*	*2*	***P***	Rail-based intermodal system with rail-post haulage
4	***P***	*2*	*2*	*1*	*0*	Rail -based intermodal system with rail-pre haulage
5	***P***	*2*	*2*	*2*	***P***	Rail-based intermodal system with rail-pre/post haulage
6	*0*	*1*	*3*	*1*	*0*	Conventional Short sea-based intermodal system
7	*0*	*1*	*3*	*2*	***P***	Short sea-based intermodal system with rail-post haulage
8	***P***	*2*	*3*	*1*	*0*	Short sea-based intermodal system with rail-pre haulage
9	***P***	*2*	*3*	*2*	***P***	Short sea intermodal system with rail-pre/post haulage

1, ***2*** and ***3*** indicate truck, rail, and vessel, respectively.
P in the penalty columns indicates that extra trucking costs are required for handling initial collections or final distributions.

Fig. 2 Description of feasible intermodal choice sets

$$\text{Minimize } TC_{ij} = \sum_{(i,j)\in A} C_{ij}X_{ij} \qquad (1)$$

where A is a set of arcs between nodes i and j; C_{ij} is a unit cost for arc (i,j) (€/TEU[4]); X_{ij} is a decision variable for arc (i,j) (TEU).

Unit cost (C_{ij}) generally plays a key role in this kind of problems. Skorin-Kapov et al. [24], for example, developed a model with a fixed constant discount factor between hubs to attempt to describe ESQ in the hub location problem as follows:

$$\text{Minimize } TC^k_{lm} = \sum_{(i,j)\in A} \sum_{(l,m)\in H} \sum_{k\in K} \alpha_{lm} \times C^k \times X^k_{ij} \times d^k_{lm} \qquad (2)$$

[4] TEU is Twenty-foot Equivalent Unit; containers or swap bodies are used as common loading unit in multimodal/multimodal freight operations due to the simplicity of transhipment.

where TC^k_{lm} is the total cost between hubs $(l,m) \in$ H, for mode k (€); α_{lm} is a discount factor between hubs $(l,m) \in$ H; C^k is the *constant* unit cost of flows between hubs $(l,m) \in$ H (€/TEU km); X^k_{ij} is the quantity shipped between $(i, j) \in$ A (TEU); d^k_{lm} is the distance between hubs $(l,m) \in$ H (km)

C^k is not a function of quantity or distance but a constant depending on mode (k). If this constant were estimated with consideration of economies of scale, this model would have taken into account only ESQ at most that indirectly affect inter-hub flows. ESQ is not gained constantly between hubs. ESQ is gain because flows are concentrated on between hubs, and accordingly the marginal cost to ship a unit is reduced due to the increase of quantity. Also, the relationship between the marginal cost and the quantity is non-linear in most logistics cases. Thus, strictly speaking, it is incorrect to assume a fixed constant cost function multiplying a discount factor between hubs (α_{lm} in Eq. 2) regardless quantity, distance, and vehicle size. O'Kelly and Bryan [20] developed a network design problem considering ESQ for passenger transports. They also assumed that economies of scale were gained in inter-hub links only. Racunica and Wynter [22] overcame this assumption by allowing the amount of economies of scale on inter-hub links to be relatively larger than other local links (i.e., drayage or pre-/post-haulage). The simplified cost formulation adopted by the above studies is:

$$TC^k_{lm} = \sum_{(i,j)\in A} \sum_{(l,m)\in H} \left(C^k_{lm}(X^k_{ij}, \alpha_{lm}) \times X^k_{ij} \times d^k_{lm} \right) \tag{3}$$

where $C^k_{lm}(X^k_{ij}, \alpha^k_{lm})$ is the unit cost function (€/TEU km or €/ton km) of k mode via hubs l and m where cost is **dependent** on flows X^k_{ij}.

The core of this approach was to develop the discount function $(C^k_{lm}(X^k_{ij}, \alpha^k_{lm}))$ which depends on quantity and the characteristics of the route between hubs.

3.2 Proposed unit cost function incorporating economies of scale in OR problem

Although one can develop a demand-dependent cost function (i.e., $C^k_{lm}(X^k_{ij}, \alpha^k_{lm})$), it is still independent of two important factors contributing unit cost reductions: distance and vehicle size. As discussed in Jara-Díaz et al. [13], both distance and quantity non-linearly affect the marginal cost. Therefore, it would be better to develop a unit cost function incorporating the dependencies associated with ESQ, ESD, and ESVS. Specifically, some freight modes might be quantity-sensitive while others might be distance-sensitive (e.g., trucks) or vehicle-size sensitive (e.g., waterborne transport). Since a multimodal freight system consists of more than two modes, the cost function might be inappropriate under ESQ only (i.e., ignoring ESD and ESVS).

Reflecting the above-mentioned issues, the objective function with the proposed unit cost function in a general form is:

$$TC^k_{ij} = \sum_{(i,j)\in A} \sum_{k\in K} \sum_{v\in V} \left(C^k_{ij}(X^k_{ij}, d^k_{ij}, S^{kv}) \times X^k_{ij} \times d^k_{ij} \right) \tag{4}$$

where $C^k_{ij}(X^k_{ij}, d^k_{ij}, S^{kv})$ is the proposed *minimum unit cost* which is a function of quantity (X^k_{ij}), distance (d^k_{ij}), and vehicle size (S^{kv}) for v type of vehicle ($|v|$ = the number of vehicle type) for each mode (k).

Generally, there would be several *unit costs* depending on the types of vehicles being used. Among them, the *minimum unit cost* of k mode can be chosen as the *minimum* value. The minimum unit cost, $C^k_{ij}(X^k_{ij}, d^k_{ij}, S^{kv})$, will be shown in the case study (Table 1) based on previous studies where 3 types of vehicles are used for each mode [4, 11, 12]. In addition, if the demand shipped is greater than the capacity of the largest vehicle, multiple vehicles need to be used. In other words, different unit cost needs to be used depending on batch strategy, which is considered here in order to specify ESVS. A simple algorithm for finding the minimum unit cost through the batch strategy is as follows:

Step 1: For each mode, set up initial given distances (d^k_{ij}) and v types of vehicle.
Step 2: Generate choice sets based on S^{kv} for each k. The number of choice sets is $2^{|v|} - 1$. For example, if there are 3 types of vehicle (i.e., $|v| = 3$) indicating ***a***, ***b***, and ***c*** (***a*** is the largest and accordingly the most efficient if it is fully loaded), the number of choice sets (i.e., batch strategy) is $2^{|3|} - 1 = 7$; (***a***→***b***→***c***), (***a***→***b***), (***a***→***c***), (***b***→***c***), (***a*** only), (***b*** only), and (***c*** only).
Step 3: For each vehicle size and mode, estimate $C^k_{ij}(X^k_{ij}, d^k_{ij}, S^{kv})$ as X^k_{ij} increases.
Step 4: For each mode and each generated choice set, calculate the number of vehicles used and the number of remaining TEUs for one smaller level vehicle.
Step 5: For each mode, estimate $\Phi^k(X, d, C_n)$, where $\Phi^k(X, d, C_n)$ is the unit cost function of nth combination.
Step 6: Find the minimum unit cost and the optimal batch for given quantity (X) and distance (d).
Step 7: Increase the fixed distance up to a certain level and return to Step 3.

Figure 3 shows an example of the unit cost competition of three different freight systems based on Table 1. When Fig. 3 is drawn, some details are assumed as follows:

- There are three different sized vehicles for each mode (i.e. seven choice sets (i.e., batch strategy))

Table 1 Unit cost function by mode and size of vehicles: $C^k(X, d^k, S^{kv})$

k	S^V	Unit cost function: $C^k(X, d^k, S^{kv})$
k = 1	Basic formulation [11, 12]	Cost (€/vehicle-km)[a] $= \alpha \times (d^1)^{\beta}$
		where α = 5.46, β = −0.278
		$C^1(X, d^1, S^{11}) = \gamma^1 \times (\frac{5.46}{2} \times (d^1)^{0.278})$ (€/TEU km)
		$C^1(X, d^1, S^{12}) = \gamma^2 \times (5.46 \times (d^1)^{0.278})$ (€/2TEU km)
		$C^1(X, d^1, S^{13}) = \gamma^3 \times (\frac{5.46}{2} \times 2.5 \times (d^1)^{0.278})$ (€/2.5TEU km)
		where γ^1, γ^2, and γ^3 are weight factors assumed as 1.0, 1.1 and 1.2, respectively
	v = 1 S^{11} = 1 TEU	$C^1(X, d^1, S^{11}) = 1.2 \times \frac{\alpha}{2} \times (d_{ij})^{\beta}$(€/TEU km)
	v = 2 S^{12} = 2 TEU	$C^1(X, d^1, S^{12}) = \frac{\alpha}{2} \times (d_{ij})^{\beta}$(€/TEU km)
	v = 3 S^{13} = 2.5 TEU	$C^1(X, d^1, S^{13}) = 0.9 \times \frac{\alpha}{2} \times (d_{ij})^{\beta}$ (€/TEU km)
k = 2	Basic formulation [11, 12]	Cost (€/km ton) $= \alpha \times (W^v \times d^2)^{\beta}$
		where,
		α = 0.58, β = 0.74
		W is the total weight of a train;
		$W^v = W_0^v$(locomotive weight) + W_1^v(flatcars weight) + X_{ij}*14.3(loads weight) for v type of train.
		$C^k(X, d^k, S^{kv}) = [0.58 \times (W_0^v + W_1^v + X * 14.3) \times d^2)^{0.74}]/X$ (€/TEU km)
	v = 1 S^{21} = 60TEU; 1 locomotive with 20 railcars	$C^2(X, d^2, S^{21}) = [0.58 \times \{(89 + 20 \times 24 + X \times 14.3) \times d^2\}^{0.74}]/X$
	v = 2 S^{22} = 75TEU; 1 locomotive with 25 railcars	$C^2(X, d^2, S^{22}) = [0.58 \times \{(89 + 25 \times 24 + X \times 14.3) \times d^2\}^{0.74}]/X$
	v = 3 S^{23} = 144 TEU; 2 locomotives with 48 rail cars	$C^2(X, d^2, S^{23}) = [0.58 \times \{(89 \times 2 + 48 \times 24 + X_i \times 14.3) \times d^2\}^{0.74}]/X$
k = 3	Basic formulation [5]	Cost (€/TEU km) = a constant for ship size
	v = 1 S^{31} = 200 TEU vessel	$C^3(X, d^3, S^{31}) = 0.08$ (US \$/TEU mile)
		≈ 0.8[b] $\times 0.08 \times (1/1.609) = 0.04$ (€/TEU km)
	v = 2 S^{32} = 500 TEU vessel	$C^3(X, d^3, S^{32}) = 0.05$ (US \$/TEU mile)
		$\approx 0.8 \times 0.05 \times (1/1.609) = 0.025$ (€/TEU km)
	v = 3 S^{33} = 800 TEU vessel	$C^3(X, d^3, S^{33}) = 0.034$ (US \$/TEU mile)
		$\approx 0.8 \times 0.04 \times (1/1.609) = 0.02$ (€/TEU km)

[a] Assumed that a vehicle is capable of carrying 2 TEUs: either two 20-foot containers or one 40-foot container

1 USD = 0.74 € in May 2009

- The vehicle types used in the test were 1 TEU, 2 TEU, and 2.5 TEU trucks; 60 TEU, 75 TEU, and 144 TEU trains; and 200 TEU, 500 TEU, and 800 TEU container ships.
- The distance travelled is 1,000 km for a long-haulage and 50 km for a drayage (Note, this assumption will be released in the case study).

Among several unit costs depending on batch strategies that every freight system has, only the minimum unit costs for the three systems are shown in Fig. 3. Curves are generally broken when batch strategy is changed.[5] The minimum unit cost is sharply dropped when batch is changed. In a vessel based multimodal system, it makes sense that the minimum unit cost is found when a 200 TEU vessel is used until 200 TEU. After 200 TEU, there would be competition between two 200 TEU vessels and one 500 TEU vessel. Either way, the unit cost jumps up. Comparing with other systems, it shows cost competitiveness after about 110 TEUs regardless of the batch strategy (i.e., the minimum cost of the vessel-based multimodal system is the minimum compared to the other systems). For rail-based multimodal system, the batch strategy changes the minimum costs seven times between the segments, indicated as A, B, C, D, E, F, and G in Fig. 3. In segments A, B, and C, single operation of a 60 TEU train, of a 75 TEU train, and of a 144 TEU train were the minimum, respectively. It is interesting to see that single operation of the 144 TEU train was more competitive than any combination of 60 and 75 TEU trains (somewhere in segment C) until reaching 144 TEU. In segment D (between 144 TEU and 150 TEU), two 75 TEU train trips showed the minimum unit cost. The

[5] The cost function of truck-only system looks a wave. However, the actually shape is also broken like others when truck-batch is changed.

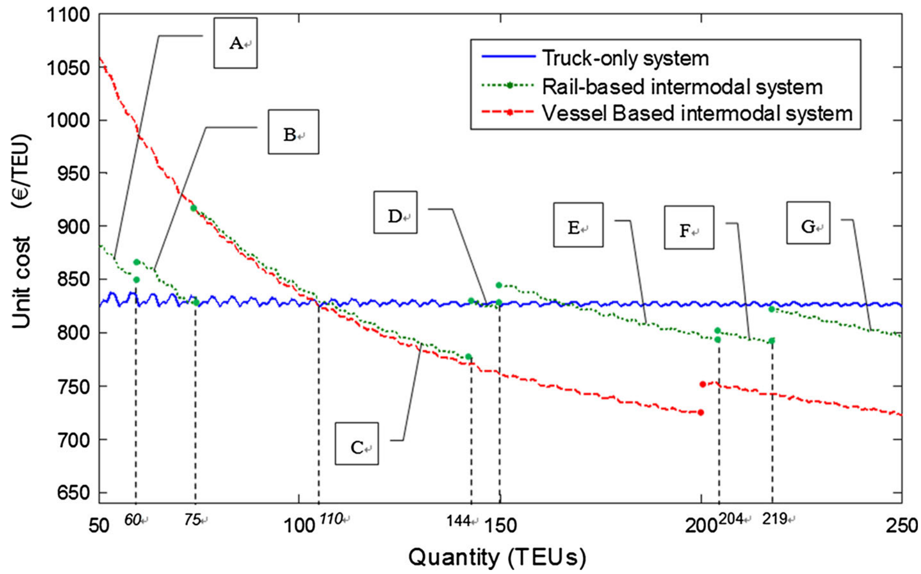


Fig. 3 Comparison of minimum unit cost as quantity increases

combination of a 144 TEU train and a 60 TEU train was found optimal in segment E (i.e., up to 204 TEUs of quantity). In segment F, the combination of a 144 TEU train and a 75 TEU train was optimal (i.e., up to 219 TEUs of quantity). From 219 TEUs of quantity, two 144 TEU trains showed the minimum unit cost. The non-continuous fluctuations of such graphs[6] give the insight that many local minima in MMCFP are found when several batch strategies are considered. The total number of batch strategies is dependent on $|v_k|$ and $|k|$ where v_k is the number of vehicle size for k mode and k is the number of transport mode (i.e. $|v_{\text{truck-only}}|=3$, $|v_{\text{rail-based multimodal}}|=3$, $|v_{\text{vessel-based multimodal}}|=3$, and $|k|=3$). Thus, as such variables increase, the local minima dramatically increase.

4 Multimodal minimum cost flow problem and the GA-based heuristic algorithm

4.1 Formulation of a multimodal minimum cost flow problem incorporating economies of scale

Consider a network $G = (N, A)$, where N is a set of nodes and A is a set of arcs. There are four types of N: origin, destination, hubs at the origin area, and hubs at the destination area, denoted as O, D, H_O, and H_D, respectively. The arcs are defined as a_{ij}^k, where i is the origin node, $i \in O$, j is the destination node, $j \in D$, k_n is a nth freight mode, and $k_n \in K$. Feasible routes are pre-defined as consecutive chains of individual arcs and denoted as r and specified in Fig. 2, where r indicates the different combinations of modes and accordingly routes.

An objective function is as follows:

$$Z = \text{Min} \sum_{(i,j)\in A} \sum_{r\in R} \Phi_{ij}^r X_{ij}^r \tag{5}$$

where X_{ij}^r is a container flow between i and j for r (TEU): Decision variable

Φ_{ij}^r is function of minimum unit cost between i and j for r (€/TEU)

Φ_{ij}^r is a function ruled by r which is shown in Fig. 2. Depending on which r is assigned, the function value can be estimated. Thus, Eq. 5 is non-linear and even non-continuous. Generally, Φ_{ij}^r is a function of all drayage processes by all k, long-haulage by all k, necessary transhipments, and penalties for rail drayage. Specifically,

$$\begin{aligned}\Phi_{ij}^r = {} & \delta_{initial} + TSc(truck, k_1) + C_{iho}^{k_1}(X_{iHo}^{k_1}, d_{iHo}^{k_1}, S^{kv}) \times d_{iHo}^{k_1} \\ & + TSc(k_1, k_2) + C_{hohd}^{k_2}\left(\sum_{(i,j)\in A} X_{ij}^{k_2}, d_{HoHd}^{k_2}, S^{kv}\right) \\ & \times d_{HoHd}^{k_2} + TSc(k_2, k_3) + C_{hdj}^{k_3}\left(X_{Hdj}^{k_3}, d_{Hdj}^{k_3}, S^{kv}\right) \\ & \times d_{Hdj}^{k_3} + TSc(k_3, truck) + \delta_{final}\end{aligned} \tag{6}$$

where $k_n \in K$, k_1, k_2, and k_3 are the first, second, and third modes assigned, respectively; $\delta_{initial}$ is a penalty for rail drayage; a constant penalty if k_1 is rail (i.e., when $r = 4, 5, 8$, and 9), 0 otherwise (€/TEU); δ_{final} is a penalty of rail drayage, a constant penalty if k_3 is rail (i.e., when $r = 3, 5, 7$, and 9), 0 otherwise (€/TEU); $TSc(k_1, k_2)$ is the transshipment cost between k_1 and k_2 (€/TEU) $C_{ij}^k(X_{ij}^k, d_{ij}^k, S^{kv})$ is

[6] Actually, the minimum cost of truck-only system is also non-continuous breaking at 1TEU, 2TEU, and 2.5 TEU. However, it was not expressed well in the graph.

the unit cost function embedding ESQ, ESD, and ESVS between i and j (€/TEU–km).[7]

There are four notable characteristics of Φ^r_{ij}. *First*, the unit of $C^k_{ij}(X^k_{ij}, d^k_{ij}, S^{kv})$ is €/TEU km in order to include the effect of ESD in a given network. ESD may not play a significant role in MMCFP since the distance is fixed in a given network. However, it is still worthwhile to include ESD because such cost function enable to consider the subtle trade-off between ESD gained long-distance trucking and diseconomies of scale for short distance of non-road modes such as rail and waterborne vessels. *Secondly*, the unit cost is not a fixed constant in this case but a function. The function values with the same demand (X^k_{ij}) and distance (d^k_{ij}) vary depending on how the different vehicle sizes (S^{kv}) are batched. *Thirdly*, transhipment costs between truck and rail, between rail and rail, and between truck and vessels are distinguished (EC [7]. Thus, *TSc* is a function of modes involved. *Finally*, the demand in unit cost function for long-haulage, indicated as $\sum_{(i,j)\in A} X^{k_2}_{ij}$ in$C^{k_2}_{hohd}\left(\sum_{(i,j)\in A} X^{k_2}_{ij}, d^{k_2}_{HoHd}, S^{kv}\right)$, is not for a single *OD* pair but the summation of all $X^{k_2}_{ij}$. This allows the clear description of ESQ in long-haulage by non-road modes.

There are four constraints as follows:

Constraint 1 Flow-conservation constraints (i.e., equality constraints)

$$\sum_r x^r_{ij} = D_{ij}, \quad \text{for all}(i,j) \in A$$

Constraint 2 Mode availability constraints

$$\sum_{r_k} \sum_{(i,j)\in A} x^{r_k}_{ij} \le u^k, r_k \subset r, k \in K$$

Constraint 3 Hub capacity constraints

$$\sum_{r_k} \sum_{(i,j)\in A} x^{r_k}_{ij} \le Hub^k, r_k \subset r, k \in K$$

Constraint 4 Non-negativity constraints (lower bound)

$$x^r_{ij} \ge 0 \quad \text{for all } r \text{ and } (i,j) \in A$$

where D_{ij} is a given demand between i and j (TEU); r_k is a k mode-related multimodal system (e.g., if k=1(truck), $r_k = 1$; if k=2(rail) $r_k = 2$, 3, 4, and 5; if k=3(vessel), $r_k = 6$, 7, 8, and 9); u^k is k mode availability (TEU/week); Hub^k is the capacity of hubs for transhipments for mode k (TEU/week) (e.g., if k=2, Hub^k is rail multimodal terminal capacity; if k=3, Hub^k is port capacity)

In *Constraint 2*, it is assumed that the rail-based multimodal system ($r = 2$) and the 2nd level rail-based multimodal systems ($r = 3$, 4, and 5) use the same freight train service and share the same limited capacity (i.e., train slots). The same assumption is similarly applied to four short sea shipping options (i.e., $r = 6$, 7, 8, and 9).

4.2 GA-based heuristic algorithm for solving a MMCFP

Genetic Algorithm (GA) is a powerful optimization method of finding a near-optimal solution, especially for non-linear and non-continuous functions such as the one proposed in this study. The rationale for adopting the GA for the proposed problem is that traditional linear and non-linear methods are impractical to solve the proposed problem. Specifically, due to the dependence of Φ^r_{ij} on X^k_{ij}, shown in Eq. 6 the number of feasible system/route choices for each O–D set is not simply $X^r_{ij} \times r$. The numerous combinations of route options and batch strategies in the proposed model make it more complicated than a general MCFP. The complexity of the proposed problem and the reason to use GA are demonstrated with a simple example in Appendix 1.

Although the Genetic Algorithm (GA) was not a perfect method guaranteeing the global optimal solution, as shown in Appendix 1, it is certainly a feasible method to find at least a near-optimal solution within reasonable computation time. A careful consideration should be given in applying GA is how to handle constraints in OR problem [6, 17, 19]; Rees and Koehler [23]; Sikora and Piramuthu [25]. This study attempts to handle constraints by modifying the initial population [18] and by developing penalty functions [17]. The outcome of the developed GA-based heuristic approach is a near-optimal solution for route, mode, and vehicle size choices including batch strategy. The basic idea and mechanism of the GA can be found in Holland [9]. The settings we used in this study were

- Real encoding rather than Binary encoding
- Stochastic universal sampling
- Modified simple crossover
- Dynamic mutation
- Elitism

The procedure of the algorithm highlighting initial population generation ensuring the equality constraint is specified in Appendix 2.

5 Application with numerical example

In this section, GA is applied for an example multimodal network in Europe. GA has found a near-optimal solution

[7] The unit cost function is fully estimated in Table 1 in Sect. 5.2.

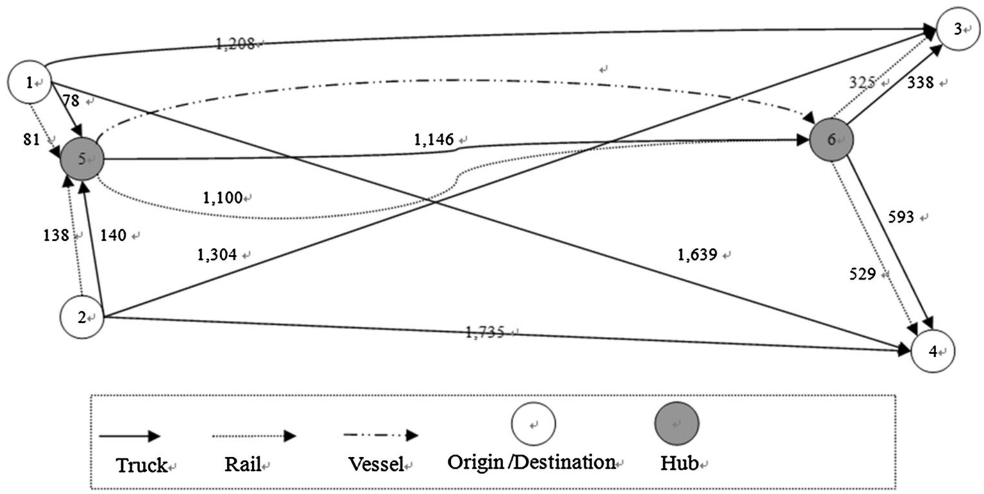


Fig. 4 Distance (d_{ij}^k) in case study area (unit: km)

in terms of system choice, route assignment, and batch strategy for given demand–capacity sets.

5.1 Study area network and OD pairs

A simplified hypothetical multimodal network is designed for testing the GA-based MMCFP. The case study area is a corridor between Western Europe and Eastern Europe with multiple modes including truck, rail and vessel. Figure 4 presents distances between six nodes and multi-modal links. Nodes 1, 2, 3, 4, 5, and 6 indicate Amsterdam, Brussels, Warsaw, Vilnius, Rotterdam, and Gdansk, respectively. The distances are estimated by using the shortest path finder of Geographic Information System (GIS). Using the node notation defined in the previous section, $O = [1, 2]$, $D = [3, 4]$, $H_O = [5]$, $H_D = [6]$.

The demands (container flows) in the OD sets were estimated based on the European Statistics Bureau (Eurostat 2008). The current container flows for $(1 \rightarrow 3)$, $(1 \rightarrow 4)$, $(2 \rightarrow 3)$, and $(2 \rightarrow 4)$ are 315, 27, 217, and 13 TEUs, respectively. The service capacity of truck, rail, and vessel are 400, 150, and 200 TEUs, respectively. The condition for navigating along the Baltic route and at the port of Rotterdam was used to reflect the capacity of vessels in service. However, since the demand and service capacity could be uncertain to some extent, we examined three more cases: current demand with unlimited capacity (Scenario 2), double demand with doubled service capacity (Scenario 3), and double demand with unlimited capacity (Scenario 4).

5.2 Cost functions incorporating economies of scale ($C_{ij}^k(X_{ij}^k, d_{ij}^k, S^{kv})$)

It is a challenging task to develop the cost functions formulated in Eq. 6. The original equations for trucks and trains were obtained from [11, 12] while those for vessels were taken from [5]. $C_{ij}^k(X_{ij}^k, d_{ij}^k, S^{kv})$ for each mode (k) is estimated by modifying these three original cost functions as presented in Table 1. Note that the cost functions in Table 1 are also used to plot Fig. 3.

5.3 Results

Table 2 shows the system/route choice and batch strategy that minimized the total logistics cost. The first column in Table 2 is 4 OD pairs (i,j). The r defined in Fig. 2 is in the second column and it is graphically specified in the third column in terms of freight transport modes between two nodes via hubs.

For Scenario 1, the system choice in (1, 3) is truck-only system ($\boldsymbol{r} = 1$) and the batch strategy was the 2.5 TEU-trucks only (C5). The main reason that all 315 TEUs are assigned to truck-only system seems to be either the limited service capacities for non-road modes (150 TEUs for rail and 200 TEUs for vessel), or the relatively long detour of the multimodal systems between Amsterdam and Warsaw (i.e. relatively short direct trucking distance). More specifically, as shown in Fig. 4, the distance of truck-only system ($\boldsymbol{r} = 1$) is 1,208 km while rail-based multimodal system ($\boldsymbol{r} = 2$) and vessel based multimodal system ($\boldsymbol{r} = 6$) are 1,516 and 2,916 km respectively. Such detours of multimodal systems crucially decrease the cost competitiveness. These longer distances of multimodal freight systems are usually compensated by ESQ gained in non-road long-haulage. However, in this case, ESQ occurred in (1, 3) does not seem to be sufficient to shift some quantity of 315 TEUs from trucks ($\boldsymbol{r} = 1$) to other multimodal system ($\boldsymbol{r} = 2$ to 9). As shown in (1, 4) and (2, 4), the quantity was not enough to achieve sufficient ESQ. If 27 TEUs in (1, 4) would be shipped by other multimodal systems. It could be successful but the ESQ was not

Table 2 Route/system choice for four scenarios: decision variables (X_{ij}^{r}) for all (i,j) and r

(i,j)	r	*Nodes and links* ①②③④: Nodes ❺❻: Hubs	Demand (TEU)	Batch Strategy: Truck pre-haulage	Train pre-haulage	Truck-only	Train long-haulage	Vessel long-haulage	Train post-haulage	Truck post-haulage
Scenario 1 (Base Scenario – Case study); Obj. value (Total Cost) = € 496,928										
(1,3)	1	① $\xrightarrow{\text{Truck}}$ ③	315			C5				
(1,4)	1	① $\xrightarrow{\text{Truck}}$ ④	27			C2				
(2,3)	3	② $\xrightarrow{\text{Truck}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	59	C2			C7		C3	
	7	② $\xrightarrow{\text{Truck}}$ ❺ $\xrightarrow{\text{Vessel}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	143	C3				C3	C3	
	8	② $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Vessel}}$ ❻ $\xrightarrow{\text{Truck}}$ ③	15		C7			C3		C5
(2,4)	1	② $\xrightarrow{\text{Truck}}$ ④	13			C1				
Scenario 2; Obj. value (Total Cost) = € 376,189										
(1,3)	1	① $\xrightarrow{\text{Truck}}$ ③	28			C1				
	5	① $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	287		C5		C5		C2	
(1,4)	1	① $\xrightarrow{\text{Truck}}$ ④	27			C2				
(2,3)	5	② $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	217		C2		C5		C2	
(2,4)	2	② $\xrightarrow{\text{Truck}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Truck}}$ ④	13	C1			C5			C1
Scenario 3; Obj. value (Total Cost) = € 877,539										
(1,3)	1	① $\xrightarrow{\text{Truck}}$ ③	630			C5				
(1,4)	5	① $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Rail}}$ ④	54		C7		C5		C5	
(2,3)	4	② $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Truck}}$ ③	40		C3		C5			C5
	9	② $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Vessel}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	393		C3			C6	C5	
(2,4)	5	② $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Rail}}$ ④	26		C3		C5		C5	
Scenario 4; Obj. value (Total Cost) = € 657,184										
(1,3)	9	① $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Vessel}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	630		C3			C2	C3	
(1,4)	3	① $\xrightarrow{\text{Truck}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Rail}}$ ④	54	C2			C5		C7	
(2,3)	7	② $\xrightarrow{\text{Truck}}$ ❺ $\xrightarrow{\text{Vessel}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	141	C3				C2	C3	
	9	② $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Vessel}}$ ❻ $\xrightarrow{\text{Rail}}$ ③	278		C3			C2	C3	
(2,4)	1	② $\xrightarrow{\text{Truck}}$ ④	1			C7				
	2	② $\xrightarrow{\text{Truck}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Truck}}$ ④	11	C3			C5			C3
	4	② $\xrightarrow{\text{Rail}}$ ❺ $\xrightarrow{\text{Rail}}$ ❻ $\xrightarrow{\text{Truck}}$ ④	14		C3		C5			C2

Truck	**Rail**	**Vessel**
C1: 2.5 TEU truck(s)* → 2.0 TEU truck(s)* → 1.0 TEU truck(s)	C1: 144-TEU train(s)* → 75-TEU train(s)* → 60-TEU train(s)	C1: 800 TEU vessel(s)* → 500 TEU vessel(s)* → 200 TEU vessel(s)
C2: 2.5 TEU truck(s)* → 2.0 TEU truck(s)	C2: 144-TEU train(s)* → 75-TEU train(s)	C2: 800 TEU vessel(s)* → 500 TEU vessel(s)
C3: 2.5 TEU truck(s)* → 1.0 TEU truck(s)	C3: 144-TEU train(s)* → 60-TEU train(s)	C3: 800 TEU vessel(s)* → 200 TEU vessel(s)
C4: 2.0 TEU truck(s)* → 1.0 TEU truck(s)	C4: 75-TEU train(s)* → 60-TEU train(s)	C4: 500 TEU vessel(s)* → 200 TEU vessel(s)
C5: 2.5 TEU truck(s) only	C5: 144-TEU train(s) only	C5: 800 TEU vessel(s) only
C6: 2.0 TEU truck(s) only	C6: 75-TEU train(s) only	C6: 500 TEU vessel(s) only
C7: 1.0 TEU truck(s) only	C7: 60-TEU train(s) only	C7: 200 TEU vessel(s) only

sufficient to overcome the detour of multimodal systems. Therefore, it makes sense that truck was the best option for both (1, 4) and (2, 4). The 217 TEUs in (2, 3) are split into the three systems. In the case of $r = 3$, the quantity shipped was almost the capacity of a 60-TEU train. In the case of $r = 7$ and 8, the total demand of vessel in (5, 6) (i.e. 193

TEUs) is also near to the capacity of a 200-TEU vessel. One may wonder why not 60 TEUs for $\boldsymbol{r} = 3$ in (2, 3) instead of 59 TEUs since it seems to be more efficient for $\boldsymbol{r} = 3$ due to a full-loaded 60-TEU train in (5, 6). Since it was not possible to check the optimal solution, we tested some suspicious candidate solutions for (2, 3). For example, we tested

- 60 TEU (i.e. full loading of a 60-TEU train) for $\boldsymbol{r} = 3$ and 142 TEU for $\boldsymbol{r} = 7$ and 15 TEU for $\boldsymbol{r} = 8$
- 60 TEU for $\boldsymbol{r} = 3$ and 144 TEU for $\boldsymbol{r} = 7$ and 12 TEU for $\boldsymbol{r} = 8$
- 60 TEU for $\boldsymbol{r} = 3$ and 144 TEU for $\boldsymbol{r} = 9$ and 12 TEU for $\boldsymbol{r} = 6$

The objective function values of those candidates was worse than the solution we found (€ 496,928).

The OD flows in Scenario 2 are same but the service capacity is increased to the infinite. The infinite service capacity indicates that non-road modes can be used without any constraints if they are more cost-effective than the truck-only system. Technically, releasing inequality constraints would result in a much larger search space for feasible solutions. This wider feasible space should lead to better solutions under these scenarios. Overall, €120,739 cost savings (€211 savings per TEU) is achieved. For arc (1, 3), 287 TEUs is shifted to a type of the multimodal system ($r = 5$). We hypothesized that the assigned 315 TEUs to the truck-only system in Scenario 1 might be caused by the limited service capacity of non-road modes and the detour in the route (1, 3). Scenario 2 clearly shows the multimodal options could be selected when service capacity increases despite of the high detours of the multimodal systems. The specific mechanism for this modal shift can be explained by the concept of consolidation at hubs. More specifically, 287 TEUs in (1, 3), 217 TEUs in (2, 3), and 13 TEUs in (2, 4) are consolidated at the hub 5 and co-shipped by a couple of trains. Also, 287 TEUs in (1, 3) and 217 TEUs in (2, 3) are consolidated at hub 6 and sent to node 3 together. These consolidations are confirmed in the batch strategy we found. For the train long-haulage, C5 is commonly found in (1, 3), (2, 3), and (2, 4): 144-TEU train only. For the train post-haulage, C2 is found.

Scenario 3 shows the impact of the double demand with the double capacity compared with Scenario 1. Overall, the total cost (objective function value) of Scenario 3 (€ 877,539) compared with that of S1 (€ 496,928) is less than double. In other words, economies of scale are more intensively gained in Scenario 3 than in Scenario 1 as quantity increases. When the service capacity constraint is released in Scenario 4, the flows in arc (1, 3) are eventually shifted to a multimodal option. It indicates the advantage of economies of scale overcomes the disadvantage of the detours. The main contribution to this huge modal shift is the consolidations at several stages (e.g. rail at node 2; vessel and rail at hub 5; rail at hub 6).

6 Concluding remarks

The significant advance in the multimodal freight transportation modelling was made by incorporate economies of scales. To our best knowledge, no attempts have been made to incorporate economies of scales in MCFP with realistic cost functions. Considering multimodality (so, MMCFP) and categorizing general economies of scale components including ESQ, ESD, and ESVS make the problem more complicated. Consideration of these into the economies of scale leads to non-linear, non-continuous, and non-convex cost functions. Given the proposed MMCFP consisted of such complicated cost functions were not effectively solved using traditional method such as linear and non-linear programming, a GA-based heuristic algorithm was applied.

The main contribution of this study was to explicitly consider several multimodal freight transport options in terms of quantity, vehicle size ($v = 1$–3), batch strategy (C1–C7), multi-modes ($k = 1$–3) and their combinations ($r = 1$–8). To date, no MCFPs generating such useful information shown in Table 2 have been formulated in a single problem. Specifically, we found system/route choice including the sequence of the modes along the route and the batch strategy for each mode selected. For simple example for highlighting the contribution of this study,

- The outcome in a type of previous studies is “truck → rail → truck”
- The outcome in another type of previous studies is “truck → rail → truck; rail is used between hubs”
- The outcome in this study is “60 TEU truck → the combination of 144 TEU train and 60 TEU train → combination of 60 TEU train and 2.5 TEU trucks 5 times; 144 TEU train and 60 TEU train should be assigned between hubs”

More specifically, it was possible to determine the multimodal and multi-batch options between hubs as well as any two nodes (i.e., origin and destination nodes). It is noted that one unit modal shift in quantity in an arc could lead to the complete change of mode/route choice since all unit costs of the entire network and batch strategy could be significantly changed. In other words, the final outcome is a consequence of (1) trade-off among ESD for long-haulage as well as diseconomies of scale with respect to drayage distance, (2) increased demand made possible to use bigger sized vehicles, which is related to both ESQ and ESVS, and (iii) the influences on the optimal batch strategy.

Nevertheless, there are some important challenges that were not fully considered in this study such as travel time, network configuration, impact of large network, and frequency of train which affect multimodal cost as well as mode/system choice [15]. Especially, since our case network is not enough, the impact of larger network should be conducted in future study.

In addition, this study is limited to clarify the individual impact of the three types of economies of scale on mode/route choice and batch strategy. This clarification would lead to an estimation of the trade-off between economies of scale obtained from long-haulage using non-road systems and diseconomies of scale due to terminal congestion/diseconomies of scale due to drayage distance. In future, to fully consider those complicated impacts of travel time, network configuration, the size of network, and frequency of train on MMCFP as well as the trade-off, a multi-objective optimization model would be developed.

Appendix 1: Rationale to use GA

In this Appendix, the complexity of the proposed problem and the reason to use GA are demonstrated with a simple example. Assume $r = 9$ feasible routes (based on Fig. 2) from node 1 to node 3 via origin hub 5 and destination hub 6. Also, consider four types of cost functions: (1) simple constant cost functions (based on Eq. 1), (2) constant hub-discount cost functions (based on Eq. 2), (3) demand-dependent hub discount cost functions (based on Eq. 3), and (4) the proposed demand-dependent cost function with multiple sized vehicle options (based on Eq. 4 or, specifically, Eq. 6). The type of problem needed to estimate the number of cases to assign might be $\sum_r X_{13}^r$ (a certain quantity) to 9 slots (where $0 \le r \le 9$, X_{ij}^r is a non-negative integer).

- When $\sum_r X_{13}^r = 1$, the four cases obviously have nine different costs for the nine options for assignment: ${}_9C_1$. The feasible assignments are

  [1,0,0,0,0,0,0,0,0], [0,1,0,0,0,0,0,0,0], …[0,0,0,0,0,0,0,1,0], [0,0,0,0,0,0,0,0,1].

- When $\sum_r X_{13}^r = 2$, the four cases have 45 different costs for the nine options for assignment: $1 \times {}_9C_1 + 1 \times {}_9C_2$. The feasible assignments are

  [2,0,0,0,0,0,0,0,0], [0,2,0,0,0,0,0,0,0], …[0,0,0,0,0,0,0,2,0], [0,0,0,0,0,0,0,0,2] when one option ($|r| = 1$) is exclusively chosen, or
  [1,1,0,0,0,0,0,0,0], [1,0,1,0,0,0,0,0,0], …[0,0,0,0,0,1,0,1], [0,0,0,0,0,0,0,1,1] when two options ($|r| = 2$) are chosen.

In the first (1) and second (2) cost functions, the total cost for all the other cases (i.e., $\sum_r X_{13}^r = \text{X}$, where X is a positive integer greater than 1) can be estimated through simple a arithmetic calculation once $\sum_r X_{13}^r = 1$ is separately estimated and saved. For example, the total cost for [1–8] can be easily estimated by multiplying 1,2,…,9 by [1,0,0,0,0,0,0,0,0], [0,1,0,0,0,0,0,0,0], …, [0,0,0,0,0,0,0,0,1], respectively. No further complexity is required. In the third (3) and fourth (4) cost functions, the total cost for $\sum_r X_{13}^r = \text{X}$ should be independently estimated. In general, as one unit of demand increases, the total costs for all r should be estimated.

- When $\sum_r X_{13}^r = 3$, the four cases have nine different costs for the 9 options for assignment: $1 \times {}_9C_1 + 2 \times {}_9C_1 + 1 \times {}_9C_1$. The feasible assignments are

  [3,0,0,0,0,0,0,0,0], [0,3,0,0,0,0,0,0,0], …[0,0,0,0,0,0,0,3,0], [0,0,0,0,0,0,0,0,3] when one option ($|r| = 1$) is exclusively chosen, or
  [2,1,0,0,0,0,0,0,0], [2,0,1,0,0,0,0,0,0], …[0,0,0,0,0,0,1,0,2], [0,0,0,0,0,0,0,1,2] when two options ($|r| = 2$) are chosen, or
  [1,1,1,0,0,0,0,0,0], [1,1,0,1,0,0,0,0,0], …[0,0,0,0,0,1,0,1,1], [0,0,0,0,0,0,1,1,1] when three options ($|r| = 3$) are chosen.

In general, the number of the routing cases for one *OD pair* is $\beta_1 \times {}_9C_1 + \beta_2 \times {}_9C_2^{\cdots} + \beta_9 \times {}_9C_9$, where β_i is the sequence number in Pascal triangles ($i = 1, 2,\ldots, 9$). Using this formula, the number of cases between any two nodes is $\sum_{i=1}^{9} {}_{\sum_{r=1}^{9} X_{ij}^r} C_{i-1} \times {}_9C_i$. The number of cases are crucially dependent on $\sum_{r=1}^{9} X_{ij}^r$. For example, when $\sum_{r=1}^{9} X_{ij}^r = 10$, 10^2, and 10^3, the number of cases for a possible route combination are 7.6^4, 3.8^{11}, and 2.6^{19}, respectively. In addition, if we take the batch strategy into account, the number of cases is increased to $\left(\sum_{i=1}^{9} {}_{\sum_{r=1}^{9} X_{ij}^r} C_{i-1} \times {}_9C_i\right) \times (2^N - 1)^K$, where N is the type of vehicle and K is the number of freight modes (see the algorithm for finding the minimum unit cost in the previous section). Furthermore, some inflows from the other nodes to hubs (e.g., X_{24}^r for any r) possibly change $\Psi_{hohd}^{k_2}\left(\sum_{(i,j)\in A} X_{ij}^{k_2}, d_{HoHd}^{k_2}, S^{kv}\right) \times d_{HoHd}^{k_2}$ (Eq. 6). For example, if 1 TEU shifts from $r = 3$ to $r = 7$ for X_{13}, it not only causes changes in the minimum unit costs (Φ_{13}^r) of the two shifted r for $r = 3$ and $r = 7$ but also changes in the minimum unit costs for all the other multimodal options (r). Therefore, the number of different cases in the function type (4) that are proposed in this study is $\left(\sum_{i=1}^{9} {}_{\sum_{r=1}^{9} X_{ij}^{r}}C_{i-1} \times {}_{9}C_{i}\right) \times (2^N - 1)^K \times OD$, where OD is the number of OD pairs in a given network. Compared to function type (3), there are obviously fewer feasible assignments than for function type (4) due to non-road drayage. If we ignore non-road drayage, $|r|$ is reduced from 9 to 3—that is, the number of cases related to non-road drayage is 6 (i.e., $r = 3, 4, 5, 7, 8$, and 9). In addition, if the batch strategy is not considered in cases with function type (3), the number of cases can be defined as $\left(\sum_{i=1}^{3} {}_{\sum_{r=1}^{3} X_{ij}^{r}}C_{i-1} \times {}_{3}C_{i}\right) \times OD$, which is significantly less than the proposed function. Unless a meta-heuristic method such as GA is used, the proposed problem might not be solvable within a reasonable amount of time.

Appendix 2: GA-based heuristic algorithm

Step 1: *Initialize* the parameters for given data such as generation number, population size, length of chromosome (which is equivalent to the number of decision variables in real-coded GA), OD demand matrix (D_{ij}), OD distance matrix d_{ij}^k for all (i,j) pairs and all modes in the cost function ($\Psi_{ij}^k(X_{ij}^k, d_{ij}^k, S^{kv})$), the lower bound (i.e., *Constraint 4*), and constant penalty ($\boldsymbol{p}$).

Step 2: *Generate* the initial population ($\tilde{X}_{ij}^r$) with two vectors: $\bar{X}_{ij}^r$ and $\hat{X}_{ij}^r$

Step 2.1: $\bar{X}_{ij}^r$ is a vector including N random real numbers, where N is the number of decision variables on arc (i,j), $0 \le \bar{X}_{ij}^r \le 1$. Note: N is determined by the number of $|r|$ and OD pairs (for example, $N = 36$ if $|r| = 9$ as in Fig. 2 and $|OD| = 4$ as in Table 2).

Step 2.2: $\hat{X}_{ij}^r$ is a vector including N random binary numbers, $\hat{X}_{ij}^r \in [0, 1]$. 1 is assigned as a component of $\hat{X}_{ij}^r$ if a random number is greater than 0.5; otherwise, 0 is assigned.

Step 2.3: The initial population ($\tilde{X}_{ij}^r$) is a vector placing $\bar{X}_{ij}^r$ and $\hat{X}_{ij}^r$ in the same raw in order.

Note: the number of raw of $\tilde{X}_{ij}^r$ is $2N$.

Step 2.4: Generate the matrix $\tilde{X}_{ij}^r$ until it reaches the maximum population size.

Note: the matrix size of $\tilde{X}_{ij}^r$ should be $2N$ multiplied by the maximum population size. It is assumed that the number of decision variables should be even.

Step 3: *Update* the initial population satisfying equality constraints (i.e., *Constraint 1*) for each arc (i,j) and generate the new population (X_{ij}^r).

$\breve{X}_{ij}^r$ is the element-wise vector multiplication for the real number side ($\bar{X}_{ij}^r$) and the binary number side ($\hat{X}_{ij}^r$) of the initial population ($\tilde{X}_{ij}^r$).

$X_{ij}^r = (\breve{X}_{ij}^r \times D_{ij} / \sum_r \breve{X}_{ij}^r)$, where $\sum_r \breve{X}_{ij}^r$ is the sum of $\breve{X}_{ij}^r$ for all r on (i,j) and D_{ij} is a given demand between i and j (TEU).

Note: a raw vector of X_{ij}^r is a candidate solution for (i,j) satisfying equality constraints and the size is N.

Step 4: *Calculate* the objective function (i.e., Eq. 5) for the population (X_{ij}^r) with $g = 1$ where g is generation number;

Save the objective function value for the gth population ($Obj(g)$).

Step 5: *Check* for inequality constraints (*Constraints 3 and 4*);

for each arc (i,j)

If $\sum_{r_k} \sum_{(i,j)\in A} x_{ij}^{r_k} \le u^k$ and $\sum_{r_k} \sum_{(i,j)\in A} x_{ij}^{r_k} \le Hub^k$

$Obj(g) = Obj(g)$

Otherwise,

$Obj(g) = Obj(g) + \text{Penalty } (p)$

Step 6: *Estimate* the fitness function.

Step 7: Increase the generation number ($g = g + 1$) and *Run* Reproduce, Crossover, Mutation, and Elitism for X_{ij}^r.

Step 8: *Return* to Step 3 if g is less than the maximum number of generations.

Steps 2 and 3 are not normally included in prototypes of the GA procedure. These two steps are designed to generate the initial population and simultaneously ensure the equality constraint. These steps would be removed if another technique to handle equality constraints could be developed. In addition, Step 7 is not fully described here. For the details of Step 7, see two pioneer studies by Holland [9].

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